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Research Article

The Markov Model in the Evolution Stability of Stock Market and Associated Fuzzy Optimization of the Investment Portfolio

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Abstract

The realistic Markov model of the management of the index of the priority investment on the base of the fuzzy mathematical programming is suggested and studied in detail. Based on the investor's higher dividend interest rate approach, there is also studied the investment portfolio capital optimization problem. The related stability problem is analyzed from the fuzzy decision making theory standpoint. The evolution stability of the market mechanism of the management of the portfolio investment capital is investigated.

Keywords: Markov Model; Market Mechanism: Evolution Stability; Investment Portfolio; Capital Optimization Problem; Fuzzy Sets; Bank Assets; Decision Theory.

Introduction

There is the whole range of models concerned with the best investment portfolio choice within riskprofit scheme based on the fuzzy decision making theory [20] which are used for the research into the market information correlation with regard to the influence on the stock market investment efficiency. In particular, it was revealed [6] that the stock market investment efficiency quadratically depends on the maximum correlation between the correlated input information and the investment package structure on the stock market. Taking into account the signicance of the evolution stock market stability, the investment portfolio capital fuzzy optimization problem becomes the most central for investors. In order to make the description of the realistic stock market behavior the most demonstrable the following article analyses the market model of assets regulated by the finite number of investment portfolio priorities with regard to total financial and share capital by means of the classical iteration-reinvestment mechanism into the fixed set of long-term deposits. The research also assumes that dividends are paid during every fixed period proportionally to the deposits according to the investments within the discrete stationary Markov process [7]. Besides, it is also believed that apart from the external common assets value growth, the portfolio investment priorities are exposed to influences caused either capital losses or capital market growth. The portfolio investment priorities aim at competition for the market capital by means of the total assets amount. The internal price formation means a certain selection market mechanism within which some portfolio priorities make the market capital whereas other lose it. The mechanism can be often adequately described on the basis of the stochastic dynamical systems theory [1, 14] whose equilibrium is provided by the distribution of share within the portfolio which is invariant with regard to the market selection process. There is also assumed that the portfolio investment priority is evolution stable if the condition under which the priority has maximum value is resistant (that is robust) to introduction of the additional priority with sufficiently low value. Our approach to the evolution stability analysis from the standpoint of its classical nancial analysis consists in maximization of the expected logarithm of growth speed of assets relative value as to the given dividends getting mechanism. However, our main objective is to analyze the growth dynamics of the stock market assets value as well as its evolution stability depending on the investment portfolio parameters within the Markov model of economic.

Thus, due to more or less lasting price making tendency on the stock market, which mainly coincides with the tendency of dividends to be paid, short periods when shares market prices considerably deviate from their set values seem to be the most important. This phenomenon known as "the market excess volatility or market variability was earlier studied by Schiller [19], and it was revealed leads to extremely irregular price dynamics modeled on the basis of the so called adaptation approaches to the stock market behavior estimation. It is the research carried out by Black and Scholes [4], who studied the short-term investment portfolio stochastic dynamics and drew a range of important conclusions about the stock market behavior prediction, which appeared to be the most signicant. Therefore, the study of the stock market model with nonhomogeneous heterogeneous distribution of the investment portfolio priorities arouses the biggest interest. It is a rationally organized [8] stock market within which the assets are evaluated by means of the relative dividend coefficient which seems to be considered as evolution stable. It is also assumed that the market chooses rational behavior strategy which, as a result, leads to the assets market efficiency. Another important issue is also a financial agent's behavior on the stock market, whose efficiency analysis has been widely researched [3, 5, 6, 8, 11-13].

Investment portfolio capitalization theory, as it is known [6, 8, 10, 15, 20], enables prediction of market agent's behavior and evaluate relevant bank assets risk. It is a priori assumed, that investors prefer higher dividend interest rate rather then lower with lower risk. Investment portfolio capital optimization problem consists in defining sufficient investment packages amounts which would provide profit that is bigger or equal to certain fuzzily determined value. As it is wished to be adequate, what is important from the standpoint of the fuzzy decision making theory, this expected value is reformulated as a problem of the analytical estimation of the fuzzy dynamical prot functional with regard to the fuzzy most possible investment capital.

Description of the Stock Market Dynamical Model

We assume, that the stock market contains the assets with long-term deposits and consumer goods, and there are $N + 1 \in Z_+$ stock and cash deposits. Each share deposit, which is normalized by unity, is supposed to get dividends at the beginning of each market period whose value will be called as $d^{(k)}_t \geq 0$, where $k=\overline{1,N}$ is a number of the share deposit and $t \in Z_+$ is a discrete time parameter. It is natural to assume that the total of paid dividends $\sum_{k=\overline{1,N}}^{N} d_t^{(k)}$ for all $t \in Z_+$, which means we do not consider the so called "dead periods" of the market activity. It is obvious that the dividends $d^{(k}_t = d^{(k)}_t[\omega_t]$, $k=\overline{1,N}$ are certain functions of the share market states S, where $[\omega_t] := \{\omega_j \in S : j=\overline{1,t}\}$. Respectively, cash deposit consists of the total amount of all dividends to be paid at the time $t \in Z_+$. Let us further assume that there exists finite number $P \in Z_+$ of the portfolio investment priorities determined by the share market investment tendency. Portfolio priorities are modeled by means of respective share indexes $\lambda^{(k)}_{t;j} = \lambda^{(k)}_{t;j}[\omega_t] \in [0; 1]$; which satisfy the normalization condition

$$\sum_{k=0}^{N} \lambda_{t,j}^{(k)} = 1 \text{ ----- (1)}$$

for all index $j=\overline{1,P}$. Thus, the portfolio investment priority $j=\overline{1,P}$ defines the following distribution of given share packages according to the relative values

$$n^{(k)}_{t;j} := \lambda^{(k)}_{t;j}\, w_{t;j}/p^{(k)}_t \text{ ----- (2)}$$

for every $k=\overline{0,N}$ where $w_{t;j} \in R_+$ is the market value of the invested j-th portfolio share, and $p^{(k)}_t \in R_+$ is a sales share price of the k-th package at the time $t \in Z_+$: The total assets of the $s^{(k)}_t \in R_+$ k-th package within the given portfolio can be defined by means of (2):

$$S_t^{(k)} := \sum_{j=1}^{P} n_{t,j}^{(k)} . \text{ ----- (3)}$$

It is easy to notice, that

$$S_t^{(k\neq 0)} = 1, S_t^{(0)} = \sum_{k=1}^{N} d_t^{(k)}, \text{ ----- (4)}$$

for our model, because the value of the k-th stock package (for $k \neq 0$) remains fixed, and the cash respectively increases by the dividends value. It leads to the following restriction on the share index of the investment priorities:

$$\sum_{k=1}^{N} \lambda_{t,j}^{(k)} \omega_{t,j} = p_t^{(k)} \text{ ----- (5)}$$

for all $k=\overline{1,N}$ and the cash price under $t = 0$ is rationed, for convenience, by unity, that is $p^{(0)}_t = 1$. On the other hand we can obtain the "budget" restriction on the relative indexes of the portfolio priorities from (2):

$$\sum_{k=0}^{N} p_t^{(k)} n_{t,j}^{(k)} = \omega_{t,j} \text{ ----- (6)}$$

for every $j=\overline{1,P}$.

Now taking into account the nonrestorable cash deposit consumption expenses, we can write the following discrete evolution equation for the total value of the j-th portfolio share:

$$w_{t+1,j} = \sum (d_{t+1,j}^{(k)} + p_{t+1}^{(k)}) n_{t,j}^{(k)} \text{ ----- (7)}$$

where the sales market price $p^{(k)}_t \in R_+$ of the *k*-th share package should be defined according to the expression (5). Based on (7) and (5) we can write the following vector evolution equation:

$$w_{t+1} = a_t(w_t) + A_t(w_t)w_{t+1}, \text{ ----- (8)}$$

where the vectors

$$w_t := \{w_{t,j} \in R : j=\overline{1,P}\}$$

$$a_t(w_t) := \left\{ \sum_{k=1}^{N} n_{t,j}^{(k)}(w_t) d_{t+1,j}^{(k)} : j = \overline{1,N} \right\},$$

and matrix

$$A_t(w_t) := \left\{ \sum_{k=1}^{N} n_{t,j}^{(k)}(w_t) \lambda_{t+1,i}^{(k)} : i,j = \overline{1,P} \right\}, \text{ ----- (9)}$$

We can consider that consumer indexes $\lambda^{(0)}_{t,j}(\omega_t) = \lambda^{(0)}$ in the frame of the model for all $t \in Z_+$ and j = 1; P are fixed. Then we can obtain from (8) that

$$\sum_{j=1}^{P}\sum_{k=1}^{N} \lambda_{t+1,j}^{(k)} w_{t+1,j} = \sum_{k=1}^{N}\sum_{j=1}^{P} d_{t,j}^{(k)} + (1-\bar{\lambda}^{(0)}) \sum_{j=1}^{P}\sum_{k=1}^{N} \lambda_{t+1,j}^{(k)} w_{t+1,j}, \text{ ---(10)}$$

that is

$$W_{t+1} = D_{t+1} / \overline{\lambda}^{(0)}, \text{ ----- (11)}$$

where we defined for all $t \in Z_+$ the general values of the investment packages and cumulative dividend as

$$W_{t+1} := \sum_{j=1}^{P}\sum_{k=1}^{N} \lambda_{t+1,j}^{(k)} w_{t+1,j}, D_{t+1} := \sum_{j=1}^{P}\sum_{k=1}^{N} \lambda_{t+1,j}^{(k)} d_{t+1,j}^{(k)} \text{ ---(12)}$$

Taking into account (11), we can obtain the expression for the economic growth (decline) cofficient

$$k_t := D_{t+1} / W_t = \overline{\lambda}^{(0)} W_{t+1} / W_t \text{ ----- (13)}$$

In the same way we can nd the market share of the j-th investment priority as

$$r_{t,j} := w_{t,j} / W_t = \frac{\overline{\lambda}(0)}{D_t} w_{t,j} \text{ ----- (14)}$$

for every $j = \overline{1,P}$. The last expression characterizes dependence between the value of the investment package and the common value of the obtained profit.

Evolution Stability Analysis

The evolution equation (8) describes the dynamics of the investment priorities packages value dynamics, which we can write as a deterministic relation

$$w_{t+1} = [1 - A_t(w_t)]^{-1} a_t(w_t) \text{ ----- (15)}$$

under condition that the determinant $det[1 - A_t(w_t)] \neq 0$. The latter condition will be satised [15], when inequality

$$|1 - A_t(w_t)_{jj}| > \sum_{i \neq j} |1 - A_t(w_t)_{ij}| \text{ ----- (16)}$$

holds for all $j = \overline{1,P}$. Considering matrix $A_t(w_t) \in End E^P$ explicit expression in (9), we can find out that the inequality (16) holds if for certain $\bar{j} \in \overline{1,P}$ the condition $w_{o,\bar{j}} > 0$ holds. Therefore, we can write down the equality (15) as

$$w_{t+1} = f_t(w_t) := [1 - A_t(w_t)]^{-1}, \text{----- (17)}$$

where the mapping $f_t : E^P \to E^P$ is algebraic.

Since we are interested in evolution stability of economy, we must find such fixed points of the mapping (17) $\bar{w}_{to}$ 2 E^P which

$$f_{to}(w_{to}) = w_{to} \text{ ----- (18)}$$

for certain $t_o \in Z_+$, and nd out their stability with respect to the small perturbations of the investment packages values.The following proposition is true.

Proposition 1. There is such set of portfolio priorities investment indexes which evolution system is stable for.

Proof. The proof is based on the existence of negative Lyapunov indexes for a small perturbation of the discrete dynamical system (17). Let us put $\tilde{w} := w_{to} + z_{to} \in E^P$ for the infinitesimal parameter $\epsilon \to 0$. Then it is easy to nd that the vector $z_{to} \in E^P$ satises the following linear iteration equation

$$z_{to+1} = f'_{to}(w_{to}) z_{to} \text{ ----- (19)}$$

with the constant matrix $f'_{to}(w_{to}) = \partial f_{to}(w_{to}) / \partial w_{to} \in End(E^P)$. Since the mapping $f_{to} : E^P \to E^P$ is algebraic and is defined by the explicit expression (17), we can find the determinant sign

$$sign \det f'_{to}(w_{to}) = (-1)^P. \text{ ----- (20)}$$

Since the relation (20) has to hold for all eigenvalues (Lyapunov indexes) of the matrix $f'_{to}(w_{to}) \in End(E^P)$ to be negative, due to the continuity of the mapping $f_t : E^P \to E^P$ we make sure that there are such portfolio indexes $\lambda^{(k)}_{to,j} \in [0; 1]$, $j = \overline{1,P}, k = \overline{0,N}$, for which the corresponding fixed point $w_{to} \in E^P$ is stable, thus proving th proposition.

Investment Portfolio Fuzzy Optimization

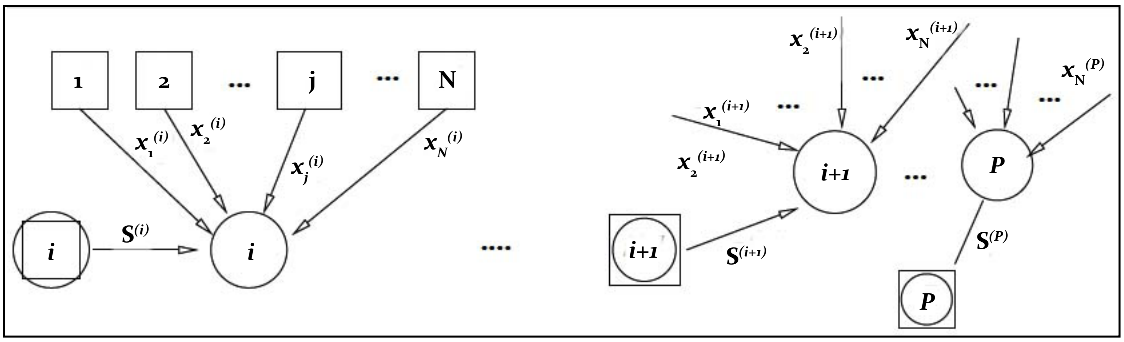
Let us consider the evolution stable share market model with a fixed set of priority investment indexes with our task being to provide aggregate portfolio optimum as to the given set of investment strategies. Thus, we need to define the respective competitive strategies interaction scheme for $j = \overline{1,P}$ investment packages. Figure 1 shows the corresponding investment portfolio structure.

Let us be given the following aggregate portfolio value under condition of evolution equilibrium:

$$W_{t_0}(\lambda_{t_0}) = \sum_{k=1}^{N}\sum_{j=1}^{P} w_{t_0,j} \lambda_{t_0,j}^{(k)} \text{ ----- (21)}$$

where the vector equilibrium value $w_{to} \in E^P$ satisfies the relation

Figure 1. Investment portfolio.



(18). Then the task of the investment portfolio optimization is to find the optimum value (21) when

$$\underset{\lambda_{t_0} \in Hom(E^P;E^N)}{\arg\max} \; W_{t_0}(\lambda_{t_0}) = \overline{\lambda}_{t_o} \text{ ----- (22)}$$

and when the following conditions

$$\left\{ \sum_{k=0}^{N} \lambda_{t_{0,j}}^{(k)} = 1 : \lambda_{t_{0,j}}^{(k)} \in [0,1], k = \overline{0,N}, j = \overline{1,P} \right\}, \text{ ----- (23)}$$

$$a_{t_0}(w_{t_0}) = [1 - A_{t_0}(w_{t_0})] w_{t_0}$$

hold. Since the matrix elements values $\bar{\lambda}_{t_o} \in Hom(E^P; E^N)$ are fuzzily defined by definition, the realistic optimization problem (22) needs to be formulated by means of fuzzy sets theory concepts [10, 16, 18] as well as fuzzy mathematic programming theory elements. For the purpose of clearer and more reasonable application of the facts required of the fuzzy mathematic programming theory adjusted to our problem (22) and (23), let us present its main elements below.

As a rule, the fuzzy set is defined as some mapping of the given set X into the lattice L:

$f: X \to L.$ ----- (24)

For all $\alpha \in L$ we can define the α-level subset of the fuzzy mapping (24) as

$N_\alpha(f) := \{x \in X : f(x) \geq \alpha\}$ ----- (25)

We can build its characteristic function $f_\alpha : X \to L$ by means of (25):

$$f^{(\alpha)}(x) ;= \begin{cases} 1 : x \in N_\alpha(f), \\ 0 : x \notin N_\alpha(f). \end{cases} \text{ ----- (26)}$$

We denote set of all fuzzy sets as $F_L(X)$. The following lemma [16] is useful.

Lemma 2 Let us $f \in F_L(X)$. Then the following representation holds:

$$f = \underset{\alpha \in L}{\vee} (\alpha \wedge f^{(\alpha)}), \text{ ----- (27)}$$

where the operation V denotes supremum, and Λ denotes inmum in the lattice L.

Proof. The proof easily follows from (26) and from the following chain of the identities:

$$\begin{aligned} &\underset{\alpha \in L}{\vee} (\alpha \wedge f^{(\alpha)}(x_0)) = [\underset{x_0 \in N_\alpha(f)}{\vee} (\alpha \wedge f^{(\alpha)}(x_0)] \vee [\underset{x_0 \notin N_\alpha(f)}{\vee} (\alpha \wedge f^{(\alpha)}(x_0)] = \\ &\underset{x_0 \in N_\alpha(f)}{\vee} (\alpha \wedge 1) = \underset{\alpha \leq f(x_0)}{\vee} \alpha = f(x_0) \end{aligned} \text{ ----- (28)}$$

for every $x_o \in X$.

We assume that L is a full distributive lattice, that is for all $\alpha \in L$ and the subset $A \subset L$ then it is follows from the condition $\alpha < supA$ there exists such $\beta \in supA$, that $\alpha \leq \beta \in supA$.

Let us consider the following standard problem of the fuzzy mathematical programming [2] with the clearly defined objective function: the objective function $\varphi : X \to R$ is given on the set X with some $N \in Z_+$ restrictions $f_j : X \to L;\ j = \overline{1,N}$, from the class of fuzzy sets $F_L(X)$. Then the problem of fuzzy mathematical programming turns into a problem of finding the optimum

$$\arg \sup_{D^{(N)}_f : x \in X} \varphi(x) = \bar{x} \in X, \text{ ----- (29)}$$

where $D_f^{(N)} := f = \overset{N}{\underset{j=1}{\wedge}} f_j$ is a fuzzy given restriction, and the corresponding level of its fuzziness $\mu(\varphi): X \to L$ It is revealed that under some assumptions the problem (29) is equivalent to the problem

$$\arg \sup_{x \in A \subset X} \varphi(x) = \bar{x} \in A, \text{ ----- (30)}$$

for some subset $A \subset X$, where the adequate fuzzy set $\mu_{(\varphi)}: R \to L$ which characterizes the level of fuzziness value of obtained optimum (30) is built.

For this purpose we will build the associated set for any $\lambda \in L$, for which the set $N_\lambda(f) \neq \phi$,

$$N(\lambda;\varphi) := \{x \in X : \varphi(x) := \sup_{y \in N_\lambda(\varphi)} \varphi(y)\}. \text{ ----- (31)}$$

Then we will mean the optimum value (29) with the fuzzy set

$$\mu(x;\varphi) := \begin{cases} \sup\limits_{x \in N(\lambda;\varphi)} \lambda & x \in \underset{\lambda \in L}{\cup} N(\lambda;\varphi); \\ 0, & x \notin \underset{\lambda \in L}{\cup} N(\lambda;\varphi). \end{cases} \text{ ----- (32)}$$

by the solution of our problem. Since obtaining the function (32) is complicated, we will find its alternative representation by means of a new fuzzy set $\mu_\varphi : R \to L$. Let us initially note, that the following equality:

$$\mu(x;\varphi) := \begin{cases} f(x) & x \in \underset{\lambda \in L}{\cup} N(\lambda;f); \\ 0, & x \notin \underset{\lambda \in L}{\cup} N(\lambda;f), \end{cases} \text{ ----- (33)}$$

holds, because from the condition $x \in supp\ \mu(\varphi) \subset X$ follows that $\mu(x;\varphi) = f(x)$. Using the relation (32) let us define the following fuzzy set $\mu_\varphi : R \to L$ as a mapping

$$\mu_\varphi(r) := \sup_{x \in \varphi^{-1}(r)} \mu(x;\varphi) = \sup_{x \in \varphi^{-1}(r)} \sup\{\lambda \in L : x \in N_\lambda(f)\} \text{ ----- (34)}$$

for every $r \in R$.

The fuzzy set (34) defines the fuzziness level of the obtained optimum (29) and satises the following natural relation:

$$\mu_\varphi(r) := \sup_{x \in \varphi^{-1}(r)} \mu(\varphi) = \sup_{x \in \varphi^{-1}(r)} f(x) \text{ ----- (35)}$$

for all $x \in X$, and the property of monotonicity:

$\mu_\varphi(r_1) \geq \mu_\varphi(r_2)$ ----- (36)

for all $r_1 \leq r_2 \in R$.

The following lemma enables to substitute the supremum problem (29) for some associated supremum problem in the lattice L.

Lemma 3 *The following representation*:

$$\sup_{D_f^{(N)} : x \in X} \varphi(x) = \sup\{r : \mu_\varphi(r) = \lambda\} \text{ ----- (37)}$$

holds for every $\lambda \in L$:

Proof. Let us provide complete proof since it is instructive. Using the lemma 2 we have:

$$\sup_{D_f^{(N)}:x\in X} \varphi(x) = \sup_{x\in X}\{\varphi(x) : x \in N\lambda(f), \lambda \in L\}$$
$$\sup_{x\in X}\{\varphi(x) : \mu(x;\varphi) = \lambda \in L\}. \quad \text{----- (38)}$$

Since the value

$$\sup_{x\in X}\{\varphi(x) : \mu(x;\varphi) = \lambda \in L\} = \sup(r : \mu_\varphi(r) = \lambda \in L), \quad \text{----- (39)}$$

then we obtain from the equalities (38), (39) and denition (34)

$$\sup_{D_f^{(N)}:x\in X} \varphi(x) = \sup\{r : \mu_\varphi(r) = \lambda, x \in X\} \quad \text{----- (40)}$$

that is the result (37).

Now we can build the optimal solution of our fuzzy mathematical programming problem (22) and (23), if we define the following values: an objective function

$$\varphi(x) := < xw_{t_0}(x), e > E^N, \quad \text{----- (41)}$$

defined in the space Hom(E^P ; E^N) R^P := X, and the corresponding fuzzy restriction sets:

$$f_j(x) := \frac{< xe_j, e > E^N - 1 + \alpha_j^{(-)} - x_j^{(0)}}{\alpha_j^{(-)} + \alpha_j^{(+)}} \in L, \quad \text{----- (42)}$$

and

$$\overline{f}(x) := < [f_{t_0}(w_{t_0}(x)) - w_{t_0}(x)], q > E^P, \quad \text{----- (43)}$$

where the lattice L := [0; 1], vectors e := (1, 1, …., 1)^T ϵ E^N and e_j := (0, 0, ….1$_j$, 0, …, 0)^T ϵ E^P, numbers $\alpha^{(-)}_j < \alpha^{(+)}_j \in R_+$, $j=\overline{1,N}$, are the fuzziness parameters of the normalizing condition and q ϵ E^P is a vector of Lagrange multipliers which satisfy the condition (18). To take into account the fuzzy restriction sets (42) let us introduce the additional vector of Lagrange multipliers $\bar{q}$ ϵ E^P which satises the condition

$$< x\tilde{q}, e > E^N - < \tilde{q}, \tilde{a} > E^P = 0, \quad \text{----- (44)}$$

where vectors

$$\tilde{a} := \alpha^{(-)} + \hat{\alpha}(\alpha^{(-)} + \alpha^{(+)}) - \tilde{e} - x^{(0)} \in E^N,$$
$$\tilde{e} := (1,1,\ldots,1)^T \in E^P, x^{(0)} := (x_1^{(0)}, x_2^{(0)}, \ldots, x_P^{(0)})^T \in E^P, \quad \text{----- (45)}$$

and diagonal matrix

$$\hat{\alpha} := \{\alpha_j \delta_{js} : \alpha_j \in [0,1], j, s = \overline{1,P}\}. \quad \text{----- (46)}$$

Then using the lemma 3 we obtain the following equation which depends on the parameter $\bar{\alpha}$:= (α$_1$, α$_2$, …., α$_p$)^T ϵ L^P:

$$(w_{t_0}(x) + \tilde{q}) \otimes e + < x\partial w_{t_0}(x)/\partial x, e > E^N + < q, \partial f_{t_0}(w_{t_0}(x))/\partial x > E^P +$$
$$< q, [\partial f_{t_0}(w_{t_0}(x))/\partial x - 1]\partial w_{t_0}(x)/\partial x > E^P = 0, \quad \text{----- (47)}$$

since the solution (47) has to satisfy the conditions (43) and (45), which unambiguously define the vectors of parameters ~q := ~q(~) and $\bar{q} := \bar{q}(\bar{\alpha})$ ϵ E^P. As it follows from the lemma 3 we can write down the definitive expression for the maximum of the objective function (41):

$$\sup_{D_f^{(N)}:x\in X} \varphi(x) = \sup_{x\in X}\{< xw_{t_0}(x), e > E^N : x \in N_\alpha(f), \alpha \in L\}$$
$$= \sup_{x\in X}\{< \bar{x}(\tilde{\alpha}) w_{t_0}(\bar{x}(\tilde{\alpha})), e > E^N : \mu(x;\varphi) = \alpha \in L\} \quad \text{----- (48)}$$
$$= \sup\{r : \mu_\varphi(r) = \lambda, \}$$

where the matrix $\bar{x}(\bar{\alpha})$ ϵ Hom(E^P ; E^N) is a solution of the differential-algebraic relations (43), (44) and (41) with the fixed parameter $\bar{\alpha}$ ϵ L^P. Now the final optimal value of the objective function (48) is

$$\sup_{D_f^{(N)}:x\in X} \varphi(x) = \overline{\varphi}(\overline{x}(\overline{\alpha})), \quad \text{----- (49)}$$

where we assume that the vector $\bar{\alpha} := (\bar{\alpha}_1, \bar{\alpha}_2 \ldots, \alpha_p)^T \in [0;1]^P$ realizes the maximum of the function $\bar{\varphi}(\bar{x}(.))$:$L^P \to$ R built above and can be determined by the classical extremum condition on the model parameters:

$$\partial\overline{\varphi}(x(\tilde{\alpha}))/\partial\tilde{\alpha}_j |_{\tilde{\alpha}_j = \overline{\alpha}_j} = 0 \quad \text{----- (50)}$$

for all $j=\overline{1,N}$. The fuzziness of the obtained optimum is characterized by the value $\mu_\varphi(\bar{\varphi}(\bar{x})))$:=$\lambda(\bar{\alpha})$ ϵ L which is given by the mapping (34).

We have to notice, that the proposed above dynamical model of the share market and investment portfolio was analyzed by means of the methods of fuzzy mathematical programming under condition that the objective function does not belong to the base lattice L, that is it is not subject to the direct optimization problem in the frame of the fuzzily given model parameters. Nevertheless, there is the corresponding reformulation of these types of problems as the optimal alternatives problems for the fuzzily given model parameters which allows for more eective analytical solution. Namely, let the objective function f_0: $X \to L$ belong to the class of the fuzzy sets $F_L(X)$ too. Then it is enough to show that the following representation:

$$\sup_{D_f^{(N)}:x\in X} f_0(x) = \bigvee_{\{\alpha_j \in L: j=\overline{1,N}\}}[\alpha_1 \wedge \alpha_2 \wedge \ldots \wedge \alpha_N \wedge \sup_{x\in \bigcap_{j=1}^N N_{\alpha_j}(f_j)} f_0(x)] \quad \text{----- (51)}$$

holds.
Indeed, on the basis of the lemma 2 we have:

$$\sup_{D_f^{(N)}:x\in X} f_0(x) = \bigvee_{x\in X}[f_0(x) \wedge f_1(x) \wedge \ldots \wedge f_N(x)]$$
$$= \bigvee_{x\in X}[f_0(x)] \bigwedge_{\{\alpha_j \in L: j=\overline{1,N}\}}(\alpha_j \wedge f_j^{(\alpha_j)}(x))$$
$$= \bigvee_{\{\alpha_j \in L: j=\overline{1,N}\}}[\alpha_1 \wedge \alpha_2 \wedge \ldots \wedge \alpha_N \bigwedge_{x\in X}(f_0(x) \bigwedge_{j=1}^N f_j^{(\alpha_j)}(x))] \quad \text{-----(52)}$$
$$= \bigvee_{\{\alpha_j \in L: j=\overline{1,N}\}}[\alpha_1 \wedge \alpha_2 \wedge \ldots \wedge \alpha_N \wedge \sup_{x\in \bigcap_{j=1}^N N\alpha_j(f_j)} \varphi(x)],$$

since the value

$$\bigwedge_{x\in X}(f_0(x) \bigwedge_{j=1}^N f_j^{(\alpha_j)}(x)) = \sup_{x\in \bigcap_{j=1}^N N\alpha_j(f_j)} f_0(x), \quad \text{----- (53)}$$

which proves the result (50).

Using the representation (50), let us consider the following problem of the fuzzy mathematical programming problem: let us be given the fuzzy sets f_j; $f \in F_L(X)$ and the value

$$\sup_{D_f:x\in X} f_0(x) = \vee_{\alpha\in L}(\alpha\wedge\varphi(\alpha)), \text{ ----- } (54)$$

where the function $\varphi : L \to L$, by definition, according to the (53), is given as

$$\varphi(\alpha) := \sup_{x\in N_\alpha(f)} f_0(x) \text{ ----- } (55)$$

for every $\alpha \epsilon L$. It is obvious that the function (55) is nonincreasing, that is for any $\alpha < \beta \epsilon L$

$\varphi(\alpha) \geq \varphi(\beta)$, ----- (56)

because then $N_\alpha(f) \subseteq N_\beta(f)$.

Considering some important applications let us put that L = [0,1] $\subset R_+$ and also define the function $\psi : L \to L$ as

$\psi(\alpha) := \alpha \wedge \varphi(\alpha)$ ----- (57)

for all $\alpha \epsilon L$:

The next proposition allows to reduce the fuzzy mathematical programming problem (54) to the fixed point problem of the mapping (55).

Proposition 4 Let the function (55) be continuous. Then it has the fixed pint $\bar{\alpha} \epsilon L$, that is

$$\bar{\alpha} = \varphi(\bar{\alpha}) \text{ ----- } (58)$$

and the value

$$\sup_{D_f:x\in X} f_0(x) = \sup_{\alpha\in L}\phi(\alpha) = \vee_{\alpha\in L}(\alpha\wedge\varphi(\alpha)) = \varphi(\bar{\alpha}) = \sup_{x\in N_\alpha(f)} f_0(x). \text{ ----- } (59)$$

Proof. Indeed, on the basis of the representation (54)

$$\sup_{D_f:x\in X} f_0(x) = \vee_{\alpha\in L}(\alpha\wedge\varphi(\alpha)), \text{ ----- } (60)$$

where the function $\varphi : L \to L$ is given by the mapping (55). Since L = [0; 1], then it follows from the continuity of the function φ : [0; 1] $\to$ [0; 1] that there exists at least one fixed point $\bar{\alpha} \epsilon L$. The following inequality $\varphi(\alpha) \geq \varphi(\bar{\alpha}) = \bar{\alpha} > \alpha$ holds for any $\alpha < \bar{\alpha}$ then $\psi(\alpha) = \alpha \wedge \varphi(\alpha) = \alpha < \bar{\alpha}$, and for $\alpha > \bar{\alpha}$, analogically, $\varphi(\alpha) \leq \varphi(\bar{\alpha})$, then $\psi(\alpha) = \alpha \wedge \varphi(\alpha) = \varphi(\alpha) \leq \bar{\alpha}$, and it is easy to show that

$$\sup_{\alpha\in L}\phi(\alpha) = \vee_{\alpha\in L}(\alpha\wedge\varphi(\alpha)) = \varphi(\bar{\alpha}) = \sup_{x\in N_{\bar{\alpha}}(f)} f_0(x), \text{ ----- } (61)$$

that is equality (59).

On the basis of the proposition 4 it is easy to prove that the following theorem solves our fuzzy mathematical programming problem.

Theorem 5 *The following equality*:

$$\sup_{D_f:x\in X} f_0(x) = \sup_{x\in X} f_0(x)\wedge f(x) = \sup_{x\in A_f} f_0(x) \text{ ----- } (62)$$

holds, where the set

$A_f := \{x \epsilon X : f_o(x)\ f(x)\}$. ----- (63)

Proof. Indeed, on one hand

$$\sup_{x\in X} f_0(x)\wedge f(x) = \max\{\sup_{x\in A_f} f_0(x), \sup_{x\notin A_f} f_0(x)\} \geq \sup_{x\in A_f} f_0(x). \text{ ----- } (64)$$

On the other hand, since on the basis of the theorem 1 for any $\bar{x} \epsilon N_{\bar{\alpha}}(f)$ the following inequalities

$$f(\bar{x}) \geq \bar{\alpha} = \sup_{x\in N_{\bar{\alpha}}(f)} f_0(x) \geq f_0(\bar{x}), \text{ ----- } (65)$$

hold, then as it follows from (65) the embedding $N_{\bar{\alpha}}(f) \subseteq A_f$. It means that

$$\sup_{x\in X} f_0(x)\wedge f(x) = \sup_{x\in N_{\bar{\alpha}}(f)} f_0(x) \leq \sup_{x\in A_f} f_0(x). \text{ ----- } (66)$$

Consequently, we obtain the result (62) from the inequalities (64) and (66).

As an application of the theorem 5 the following proposition [16], which characterizes the optimal alternative choice for our fuzzy mathematical programming problem, is true.

Proposition 6 *The alternative $x \epsilon X$ is an optimal solution if and only if the vector* $\bar{y} := (\bar{x};\bar{\alpha}) \in x\times[0,1], \bar{\alpha} := \min_{j=\overline{0,N}} f_j(\bar{x})$, *is the optimum of the following classical optimization problem*:

$$\max \alpha = \bar{\alpha} \text{ ----- } (67)$$

under conditions that the inequalities

$$0 \leq \alpha \leq f_j(x) \text{ ----- } (68)$$

hold for all $j=\overline{1,N}$ and $x \epsilon X$.

The applications of the theorem 4 which are associated with our model stability analysis as well as the optimal alternatives choice analysis with regard to the investment portfolio priority indexes formation will be studied in the next research.

Conclusions

Like the majority of modern research into the problem of evolutionary stability of the market mechanism of the financial portfolio management our study is based on some initial assumptions and tenets concerned with the choice and proof of the adequate mathematical model. In particular, we claimed that the indexes of the portfolio investment priorities are completely informationally adopted as of the time when dividends are paid and are described by means of the Markov discrete process. The later does not include such priority choice strategy as purchase and sale" determined by the nature of such stock market price fluctuations as increase-decrease" etc. Another important aspect also includes the fact that in our model the shares total value within the portfolio which can be optimized according to choice of the investment priority indexes increases only due to the internal capital investments. It is obvious that this mechanism can be complemented by certain process of the external increase". Then the portfolio investment priorities indexes will increase the shares total value due to the less successful priority indexes package values. The important aspect of our model is also based on the fact that appropriately formulated problem of fuzzy mathematical programming to find the shares value optimum under evolution stability conditions considers the fuzzily controlled portfolio investment indexes. Therefore the so called market mechanism stability robustness as well as its dynamic flexibility related to the ability to control share value gains being determined by the given cash capital investment priority index within the portfolio are guaranteed.

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