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Uniform Triple Statistical Convergence of Fractional order on Time Scales Defined by Musielak-Orlicz Function

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Abstract

We introduce a triple sequence notion of uniform statistical convergence on an arbitrary time scale. However, we will γ - uniform Cauchy function on a time scale with respect of fractional order Δ^α of Musielak - Orlicz function. 2010 Mathematics Subject Classification. 40F05, 40J05, 40G05.

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Introduction

The notion of statistical convergence is closely related to the density of the subset of N. So we will define on triple sequence of γ - uniform and (λ,γ)- uniform density of the subset of the time scale. We will focus on constructing a concept of triple sequence of γ- uniform or (λ,γ)- uniform statistical convergence and γ- uniform statistical Cauchy functions on time scales of fractional order, i.e., Δ^α depends on γ and (λ,γ) respectively. We here recall some basic concepts and notations from the theory of time scales. A time scale is an arbitrary nonempty closed set of real numbers. We use the symbol T to denote a time scale. A time scale has the topology that it inherits from the real numbers with the standard topology. It allows unifying the usual differential and integral calculus for one variable. One can replace the range of definition R of the functions under consideration by an arbitrary time scale T.

The forward jump operator σ:T→T can be defined by $\sigma_{mnk} = inf\{(m_s\ n_s\ k_s) \in T:(m_s\ n_s\ k_s)>(mnk)\}$, for $(m,n,k) \in T$, and the graininess function μ:T→[0,∞] is defined by $\mu_{mnk} = \sigma_{mnk} - (mnk)$. In this definition, we put $inf\phi = supT$, where ϕ is an empty set. A half open interval on an arbitrary time scale T is given by $(a,b)_T = \{(m,n,k) \in T: a \le (m,n,k) < b\}$.

Now, let A denote the family of all left closed and right open intervals of T of the form $[a,b]_T$. Let $(m_s\ n_s\ k_s)$: A→[0,∞] be the set function on A such that $(m_s\ n_s\ k_s)([a,b]_T) = b-a$. Then, it is known that $(m_s\ n_s\ k_s)$ is a countably additive measure on A. Now, the Cara theory extension of the set function $(m_s\ n_s\ k_s)$ associated with family A is said to be the Lebesgue Δ^α- measure on T and is denoted by $\mu_{\Delta\alpha}$. In this case, it is known that if a ∈ T-{maxT}, then the single point set {a} is Δ^α- measurable and $\mu_{\bar{\Delta}^\alpha}(a) = \sigma(a) - a$. If a,b ∈ T and a ≤ b then $\mu_{\bar{\Delta}^\alpha}\left((a,b)_{\mathbb{T}}\right) = b - \sigma(a)$. If $a,b \in T - \{maxT\}, a \le b; \mu_{(\Delta^\alpha)}(a,b)_T) = \sigma(b) - \sigma(a)$ and $\mu_{\bar{\Delta}^\alpha}\left((a,b)_{\mathbb{T}}\right) = \sigma(b) - a.$. In this study, introduce the triple sequence $x=(x_{mnk})$ of γ- uniform and (λ,γ)- uniform density of a set and γ- uniform and (λ,γ)- uniform statistical convergence and some properties of γ- uniform and (λ,γ)- uniform statistical convergence on time scales. A triple sequence (real or complex) can be defined as a function x:N×N×N→R(C), where N, R and C denote the set of natural numbers, real numbers and complex numbers respectively. The different types of notions of triple sequence was introduced and investigated at the initial by [1-10].

A triple sequence $x = (x_{mnk})$ is said to be triple analytic if

$$sup_{m,n,k} \left|x_{mnk}\right|^{\frac{1}{m+n+k}} < \infty.$$

The space of all triple analytic sequences are usually denoted by Λ^3. A triple sequence $x = (x_{mnk})$ is called triple gai sequence if

$$\left((m+n+k)!\left|x_{mnk}\right|\right)^{\frac{1}{m+n+k}} \to 0 as m,n,k \to \infty.$$

The notion of difference sequence spaces (for single sequences) [11].

$$Z(\ddot{A}) = \{x = (x_k) \in w : (\ddot{A}x_k) \in Z\}$$

for Z = c, c_0 and l_∞, where $\Delta x_k = x_k - x_{k+1}$ for all k ∈ N.

The difference triple sequence [5] and is defined as

$$\Delta x_{mnk} = x_{mnk} - x_{m,n+1,k} - x_{m,n,k+1} + x_{m,n+1,k+1} - x_{m+1,n,k} + x_{m+1,n+1,k} + x_{m+1,n,k+1} - x_{m+1,n+1,k+1}$$

and $\Delta^0 x_{mnk} = \langle x_{mnk}. \rangle$

Some New Difference Triple Sequence Spaces with Fractional Order

Let Γ(*α*) denote the Euler gamma function of a real number *α*. Using the definition Γ(*α*) with α ∉ {0,-1,-2,-3,...} can be expressed as an improper integral as follows: $\Gamma(\alpha) = \int_0^\infty e^{-x} x^{\alpha-1} dx$

where α is a positive proper fraction. We have defined the generalized fractional triple sequence of difference operator

$$\Delta_\gamma^\alpha (x_{mnk}) = \Sigma_{u=0}^\infty \Sigma_{v=0}^\infty \Sigma_{w=0}^\infty \frac{(-1)^{u+v+w} \Gamma(\alpha+1)}{(u+v+w)! \Gamma(a-(u+v+w)+1)} x_{m+u,n+v,k+w}$$

In particular, we have

$$\Delta^{(1/2)}(x_{mnk}) = x_{mnk} - 1/16 x_{(m+1,n+1,k+1)} - ?\bullet\bullet\bullet.$$

$$\Delta^{-(1/2)}(x_{mnk}) = x_{mnk} + 5/16 x_{(m+1,n+1,k+1)} + ?\bullet\bullet\bullet.$$

$$\Delta^{(2/3)}(x_{mnk}) = x_{mnk} - 4/81 x_{(m+1,n+1,k+1)} - ?\bullet\bullet\bullet.$$

Now we determine the new classes of triple difference sequence $\Delta_\gamma^\alpha (x)$ as follows:

$$\Delta_\gamma^\alpha (x) = \left\{x : (x_{mnk}) \in w^3 : (\Delta_\gamma^\alpha x) \in X\right\},$$

where

$$\Delta_\gamma^\alpha (x_{mnk}) = \Sigma_{u=0}^\infty \Sigma_{v=0}^\infty \Sigma_{w=0}^\infty \frac{(-1)^{u+v+w} \Gamma(\alpha+1)}{(u+v+w)! \Gamma(a-(u+v+w)+1)} x_{m+u,n+v,k+w}$$

and $X \in \chi_f^{3\Delta}(x) = \chi_f^3(\Delta_\gamma^\alpha x_{mnk}) = \mu_{mnk}(\Delta_\gamma^\alpha x)$

$$= \left[f_{mnk}\left(\left((m+n+k)! |\Delta_\gamma^\alpha x_{mnk}| \right)^{\frac{1}{m+n+k}}, \bar{0} \right) \right].$$

Proposition 1 (i) For a proper fraction α, *Δ^α: W×W×W→W×W×W* defined by equation of (4.1) is a linear operator.

(ii) For *α,β>0*, $\Delta^\alpha\left(\Delta^\beta\left(x_{mnk}\right)\right) = \Delta^{\alpha+\beta}\left(x_{mnk}\right)$ and $\Delta^\alpha\left(\Delta^{\ \alpha}\left(x_{mnk}\right)\right) = x_{mnk}$

Proof. Omitted.

Proposition 2 For a proper fraction α and f be an Musielak-Orlicz function, if $\chi_f^3(x)$ is a linear space, then $\chi_f^{3\Delta_\gamma^\alpha}(x)$ also a linear space.

Proof. Omitted.

Definitions and Preliminaries

Throughout the article $w^3, \chi^3(\Delta), \Lambda^3(\Delta)$ denote the spaces of all, triple gai difference sequence spaces and triple analytic difference sequence spaces respectively.

Subramanian N and Esi A (2015) [8] introduced by a triple entire sequence, triple analytic sequences and triple gai sequence. The triple sequence spaces of $\chi^3(\Delta)$ and $\Lambda^3(\Delta)$ are defined as follows:

$$\chi^3(\Delta) = \left\{x \in w^3 : \left((m+n+k)! |\Delta x_{mnk}|\right)^{1/m+n+k} \to 0 as m,n,k \to \infty\right\}$$

and

$$\Lambda^3(\Delta) = \left\{x \in w^3 : sup_{m,n,k} |\Delta x_{mnk}|^{1/m+n+k} < \infty\right\}.$$

Definition 1 An Orlicz function [12] is a function *M*:[0,∞)→[0,∞) which is continuous, non-decreasing and convex with M(0)=0, M(x)>0, for x>0 and M(x)→∞ as x→∞. If convexity of Orlicz function M is replaced by M(x+y) ≤ M(x) + M(y), then this function is called modulus function. Lindenstrauss J, Tzafriri L [13] used the idea of Orlicz function to construct Orlicz sequence space.

A sequence $g = (g_{mn})$ defined by

$$g_{mn}(v) = \sup\{|v|u - (f_{mnk})(u) : u \geq 0\}, m,n,k = 1,2,\cdots$$

is called the complementary function of a Musielak-Orlicz function *f*. For a given Musielak-Orlicz function *f*, [14] the Musielak-Orlicz sequence space t_f is defined as follows

$$t_f = \left\{x \in w^3 : I_f\left(|x_{mnk}|\right)^{1/m+n+k} \to 0\right\}, \text{ as } m, n, k \to \infty$$

where I_f is a convex modular defined by

$$I_f(x) = \sum_{m=1}^\infty \sum_{n=1}^\infty \sum_{k=1}^\infty f_{mnk}\left(|x_{mnk}|\right)^{1/m+n+k}, x = (x_{mnk}) \in t_f.$$

We consider t_f equipped with the Luxemburg metric

$$d(x,y) = \sum_{m=1}^\infty \sum_{n=1}^\infty \sum_{k=1}^\infty f_{mnk}\left(\frac{|x_{mnk}|^{1/m+n+k}}{mnk}\right).$$

Definition 2 A subset E of N is said to be uniformly dense if

$u(E)=\lim_{uvw}\to\infty\frac{1}{uvw}\sum_{m=1}^{u}\sum_{n=1}^{v}\sum_{k=1}^{w}\chi_E((mnk)+\gamma)=a$ uniformly in γ or,

equivalently $\lim_{uvw\to\infty}\frac{1}{uvw}E\bigcap\{\gamma+1,\gamma+2,\cdots,\gamma+(uvw)\}|=a,$

uniformly in γ, where γ=0,1,2,... and χ_E is characteristic function.

Definition3 A triple sequence x=(x_mnk) be a real or complex valued sequence. If $\lim_{u,v,w\to\infty}\frac{1}{uvw}|\{\gamma\le(m,n,k)\le(u,v,w)+\gamma:|x_{mnk}-1|\ge\epsilon\}|=0$

uniformly in γ, triple sequence is said to be γ- uniform statistically convergent to l for ε>0.

Definition 4 Let K⊂N and define the (λ,γ)- uniform density of K by

$\Im_\lambda^\gamma(K)=\lim_{u,v,w}\to\infty\frac{1}{\lambda_{uvw}}|\{(uvw)+\gamma=\lambda_{uvw}$
$\le(m,n,k)\le(uvw)+\gamma:(m,n,k)\in K\}|.\Im_\lambda^\gamma K$ reduces to the $\Im^\gamma(K)$ in case of λ_{uvw} = uvw for all u,v,w ∈ N.

Definition 5 A triple sequence x = (x_{mnk}) is said to be (λ,γ) - uniform statistically convergent to l if

$$\lim_{u,v,w\to\infty}\frac{1}{\lambda_{uvw}}|\{(uvw)+\gamma-\lambda_{uvw}\le(uvw)+\gamma:|x_{mnk}-1|\ge\varepsilon\}|0$$

for every ε>0 uniformly in γ.

Definition 6 A triple sequence of real valued function x_{mnk}, measurable (in Lebesgue sense) on the interval (1,∞), is said to be strongly summable to l = l_x if

$$\lim_{uvw}\to\infty\frac{1}{uvw}\int_1^u\int_1^v\int_1^w|x_{mnk}-1|^p\,dmdndk=0,1\le p<\infty$$

Definition 7 Let λ∈Λ, let p be a real number, and x_{mnk} be a real valued function which is measurable (in Lebesgue sense) on the interval (1,∞), if

$$\lim_{u,v,w\to\infty}\frac{1}{\lambda_{uvw}}\int_{u-\lambda_u+1}^{u}\int_{v-\lambda_v+1}^{v}\int_{w-\lambda_w+1}^{w}|x_{mnk}-1^p\,dmdndk=0$$

Definition 8 Suppose that Ω is a Δ^α- measurable subset of T then, for (m,n,k)∈T is defined Ω(m,n,k) by Ω(m,n,k) = {(m_s n_s k_s)∈(m_0 n_0 k_0),(m,n,k)]$_T$:m_s ns k_s∈Ω}. The density of Ω on T, denoted by

$$\Im_{\mathbb{T}}(\Omega),\Im_{\mathbb{T}}(\Omega)=\lim_{m,n,k\to\infty}\frac{\mu_\Delta\alpha(\Omega)}{\mu_\Delta\alpha([(m_0n_0k_0),(mnk)]_{\mathbb{T}})}$$

provided that the above limit exists. The triple sequence X is statistically convergent to a real number 1 on T if, for every ε>0 $\Im_{\mathbb{T}}$ ({(m,n,k)∈T:|x_{mnk}-l|≥ε})=0 where X:T^3→R^3 is a Δ^α- measurable function.

Definition 9 A triple sequence X:T^3→R^3 be a Δ^α- measurable function X is statistical Cauchy on T if, for each ε > 0, there exists a number (m_1,n_1,k_1) > (m_0,n_0,k_0) ∈ T such that

$$\lim_{m,n,k\to\infty}\frac{\mu_\Delta\alpha(\{(m_sn_sk_s)\in[(m_0n_0k_0),(mnk)]_{\mathbb{T}}:|x_{m_sn_sk_s}-x_{m_1n_1k_1}|\ge\varepsilon}{\mu_\Delta\alpha([(m_0n_0k_0),(mnk)]_{\mathbb{T}})}=0.$$

Definition 10 Let α be a proper fractional order and Ω be a Δ_γ^α- measurable subset of T. Then Ω((m,n,k),γ) is defined by Ω((m,n,k),γ)={($m_s n_s k_s$)∈[γ+($m_0n_0k_0$)-1,(mnk)+γ):($m_sn_sk_s$)∈Ω}, for (m,n,k)∈T.

The γ- uniform density of Ω on T, denoted by $\Im_{\mathbb{T}}^{\text{ã}}(\Omega)$ as follows:

$$\Im_{\mathbb{T}}^{\text{ã}}(\Omega)=\lim_{m,n,k\to\infty}\frac{\mu_{\Delta_\gamma^\alpha}(\Omega((m,n,k),\gamma))}{\mu_{\Delta_\gamma^\alpha}([\gamma+(m_0n_0k_0)-1,(mnk)+\gamma]_{\mathbb{T}})},$$

provided that the above limit exists.

Definition 11 Let α be a fractional order, f be a Musielak-orlicz function and a triple sequence X:T^3→R^3 be a Δ_γ^α- measurable function. Then the triple sequence X is γ- uniform statistically convergent to a real number l on T if

$$\lim_{m,n,k\to\infty}\frac{\mu_{\Delta_\gamma^\alpha}((m_sn_sk_s)\in[\gamma+(m_0n_0k_0)-1,(mnk)+\gamma):f_{mnk}(|x_{m_sn_sks}-l|\ge\varepsilon))}{\mu_{\Delta_\gamma^\alpha}([\gamma+(m_0n_0k_0)-1,(mnk)+\gamma]_{\mathbb{T}})}=0,$$

uniformly in γ for every ε>0. In this case, we write $\hat{S}_{f\mathbb{T}}^\gamma-lim_{mnk\to\infty}X_{mnk}=l.$ The set of all γ - uniform statistically convergent functions on T will be denoted by $\hat{S}_{f\mathbb{T}}^\gamma$.

Definition 12 Let α be a fractional order, f be a Musielak-orlicz function and a triple sequence X:T^3→R^3 be a Δ_γ^α- measurable function. Then the triple sequence X is an γ- uniform statistical Cauchy function on T if there exists a number ($m_1n_1k_1$) > (m_0 n_0k_0)∈T such that

$$\lim_{m,n,k\to\infty}\frac{\mu_{\Delta_\gamma^\alpha}((m_sn_sk_s)\in[\gamma+(m_0n_0k_0)-1,(mnk)+\gamma):f_{mnk}(|x_{m_sn_sks}-l|\ge\varepsilon))}{\mu_{\Delta_\gamma^\alpha}([\gamma+(m_0n_0k_0)-1,(mnk)+\gamma]_{\mathbb{T}})}=0,$$

for each ε>0 uniformly in γ.

Definition 13 Let α be a proper fractional order and Ω((m,n,k),γ,λ) be a $\Delta_{(\lambda,\gamma)}^\alpha$- measurable subset of T, is defined by

Ω((m,n,k),γ,λ)={($m_sn_sk_s$)∈[(mnk)+γ-λ_{mnk}+($m_0n_0k_0$)-1,(mnk)+γ):($m_sn_sk_s$)∈Ω}, for (m,n,k)∈T and it is denoted by

$\Im_{\mathbb{T}}^{(\lambda,\gamma)}(\Omega)$ as follows:

$$\Im_{\mathbb{T}}^{(\lambda,\gamma)}(\Omega)=\lim_{m,n,k\to\infty}\frac{\mu_{\Delta_{(x,y)}^\alpha}(\Omega((m,n,k),\gamma,\lambda))}{\mu_{\Delta_{(x,y)}^\alpha}([(mnk)+\gamma-\lambda_{mnk}+(m_0n_0k_0)-1,(mnk)+\gamma]_{\mathbb{T}})},$$

provided that the above limit exists.

Definition 14 Let α be a fractional order, f be a Musielak-orlicz function and a triple sequence X:T^3→R^3 be a $\Delta_{(\lambda\gamma)}^\alpha$- measurable function. Then the triple sequence X is (λ,γ)- uniform statistically convergent to a real number l on T if

$$\lim_{m,n,k\to\infty} \frac{\mu_{\Delta^{\alpha}_{\gamma,\lambda}}((m_s n_s k_s)\in[(mnk)+\gamma-\lambda_{mnk}+(m_0n_0k_0)-1,(mnk)+\gamma): f_{mnk}(|x_{m_sn_sks}-l|\geq\varepsilon))}{\mu_{\Delta^{\alpha}_{\gamma}}([(mnk)+\gamma-\lambda_{mnk}+(m_0n_0k_0)-1,(mnk)+\gamma]_{\mathbb{T}})}=0,$$

uniformly in γ for every ε>0. In this case, we write

$\hat{S}^{\lambda\gamma}_{f_{\mathbb{T}}}-\lim_{m,n,k\to\infty} X_{mnk}=l.$ The set of all (λ,γ)- uniform statistically convergent functions on T will be denoted by $\hat{S}^{\lambda\gamma}_{f_{\mathbb{T}}}$

Main Results

Proposition 3 If f be a Musielak-Orlicz function and two triple sequences X,Y:T^3→R^3 with $\hat{S}^{\lambda\gamma}_{f_{\mathbb{T}}}-\lim_{m,n,k\to\infty} X_{mnk}=l_1$ and

$\hat{S}^{\lambda\gamma}_{f_{\mathbb{T}}}-\lim_{m,n,k\to\infty} X_{mnk}=l_2$ then the following statements hold:

$$(i)\ \hat{S}^{\lambda\gamma}_{f_{\mathbb{T}}}-\lim_{m,n,k\to\infty}(X_{mnk}+Y_{mnk})=l_1+l_2,$$

$$(ii)\ \hat{S}^{\lambda\gamma}_{f_{\mathbb{T}}}-\lim_{m,n,k\to\infty}(c,X_{mnk})=c,l1,(c\in\mathbb{R}).$$

Proof. Omitted.

Theorem 1 Let α be a fractional order, f be a Musielak-Orlicz function and a triple sequence X:T^3→R^3 be a $\Delta_{(\lambda\gamma)}^{\alpha}$- measurable function. Then the triple sequence X is (λ,γ) - uniform statistically convergent on T⇔ the triple sequence X_{mnk} is a (λ,γ) - uniform statistical Cauchy function on T.

Proof. This prove is similar to Theorem 3 of [15].

Theorem 2 Let α be a fractional order and $\hat{S}^{\gamma}_{\mathbb{T}}\subset\hat{S}^{\lambda,\gamma}_{\mathbb{T}}$ ⇔

$$\lim_{m,n,k\to\infty}\frac{\inf\mu_{\Delta^{\alpha}_{\gamma}}([(mnk)+\gamma-\lambda_{mnk}+(m_0n_0k_0)-1,(mnk)+\gamma)_{\mathbb{T}})}{\mu_{\Delta^{\alpha}_{\gamma}}([\gamma+m_0n_0k_0)-1,(mnk)+\gamma]_{\mathbb{T}})}>0.$$

Proof. For a given ε>0, we have

$$\mu_{\Delta^{\alpha}_{\gamma}}\left((m_sn_sk_s)\in[m_0n_0k_0]-1,(mnk)+\gamma\right)_{\mathbb{T}}: f_{mnk}(|X_{m_sn_sk_s}-1|\geq\varepsilon))\supset$$
$$\mu_{\Delta^{\alpha}_{\gamma}}\left((m_sn_sk_s)\in(mnk)+\gamma-\lambda_{mnk}+(m_0n_0k_0)-1,(mnk)+\gamma)_{\mathbb{T}}: f_{mnk}(|X_{m_sn_sk_s}-1|\geq\varepsilon)\right).$$

Therefore,

$$\mu_{\Delta^{\alpha}_{\gamma}}\left((m_sn_sk_s)\in[m_0n_0k_0]-1,(mnk)+\gamma\right)_{\mathbb{T}}: f_{mnk}(|X_{m_sn_sk_s}-l|\geq\varepsilon))\times$$
$$\mu_{\Delta^{\alpha}_{\gamma}}\left(\gamma+(m_0n_0k_0)-1,(mnk)+\gamma\right)_{\mathbb{T}})^{-1}$$

$$\geq\frac{\mu_{\Delta_{(\lambda,\gamma)}}\alpha\left((m_sn_sk_s)\in\left[(mnk)+\gamma-\lambda_{mnk}+(m_0n_0k_0)-1,(mnk)+\gamma\right]_{\mathbb{T}}: f_{mnk}\left(|X_{m_sn_sk_s}-l|\right)\geq\varepsilon\right)\right)}{\mu_{\Delta^{\alpha}_{\gamma}}\left((\gamma+(m_0n_0k_0)-1,(mnk)+\gamma\right)_{\mathbb{T}})}$$
$$=\frac{\mu_{\Delta_{(\lambda,\gamma)}}\alpha\left(\left((mnk)+\gamma-\lambda_{mnk}+(m_0n_0k_0)-1,(mnk)+\gamma\right)_{\mathbb{T}}\right)}{\mu_{\Delta^{\alpha}_{\gamma}}\left((\gamma+(m_0n_0k_0)-1,(mnk)+\gamma\right)_{\mathbb{T}})}\times$$
$$\frac{1}{\mu_{\Delta^{\alpha}_{\gamma}}\left(\left((mnk)+\gamma-\lambda_{mnk}+(m_0n_0k_0)-1,(mnk)+\gamma\right)_{\mathbb{T}}\right)}$$
$$\mu\Delta^{\alpha}_{(\lambda,\gamma)}((m_sn_sk_s)\in(mnk)+\gamma-\lambda_{mnk}+(m_0n_0k_0)-1,(mnk)+\gamma)_{\mathbb{T}}: f_{mnk}(|Xm_sn_sk_s-l|)\geq\varepsilon))$$

Hence by using (6.1) and taking the limit as m,n,k→∞, we get

$$\left(X_{m_sn_sk_s}\right)\to^{\left(\hat{s}^{(\gamma)}_{\mathbb{T}}\right)} l\Rightarrow\left(X_{m_sn_sk_s}\right)\to l\left(\hat{s}^{(\lambda,\gamma)}_{\mathbb{T}}\right).$$

Definition 15 Let α be a fractional order and a triple sequence X:T3→R^3 be a Δ^{α}- measurable function and 0<p<∞, then X is strongly p- ces`aro summable on T if there exists some l∈R such that

$$\lim_{m,n,k\to\infty}\frac{1}{\mu^{\alpha}_{\Delta}([(m_0n_0k_0),(mnk)_{\mathbb{T}}])}\int_{[(m_0n_0k_0)(mnk)_{\mathbb{T}}]}$$
$$f_{mnk}(|xm_sn_sk_s-l|^p\ \Delta^{\alpha}(xm_sn_sk_s))=0.$$

The set of all p - cesàro summable functions on T will be denoted by $[W_p]_{\mathbb{T}}^{f}$.

Definition 16 Let α be a fractional order and a triple sequence X:T^3→R^3 be a $\Delta_{(\lambda,\gamma)}^{\alpha}$- measurable function and 0<p<∞, X is said to be (λ,γ) uniformly strongly p- summable on T if there exists some l∈R such that

$$\lim_{m,n,k\to\infty}\frac{1}{\mu^{\alpha}_{\Delta(\lambda,\gamma)}([(mnk)+\gamma-\lambda mnk+(m_0n_0k_0)-1,((mnk)+\gamma)_{\mathbb{T}}])}$$

$$\int_{[(mnk)+\gamma-\lambda mnk+(m0n0k0)-1,((mnk)+\gamma)\mathbb{T}]} f_{mnk}(|x_{m_sn_sk_s}-1|^p\ \Delta\alpha(x_{m_sn_sk_s}))=0.$$

In this case, we can write $\left[\hat{W}_{\gamma p}\right]^{f}_{\mathbb{T}}-\lim X_{m_sn_sk_s}=l.$

The set all (λ,γ) uniformly strongly p- summable function on T will be denoted by $\left[\hat{W}_{\gamma p}\right]^{f}_{\mathbb{T}}$.

Lemma 1 Let α be a fractional order and a triple sequence X:T^3→R^3 be a $\Delta_{(\lambda,\gamma)}^{\alpha}$- measurable function and

Ω((m,n,k),γ,λ)={($m_s n_s k_s$)∈[(mnk)+γ-λ_{mnk}+($m_0n_0k_0$)-1,(mnk)+γ)_T:($m_s n_s k_s$)∈Ω},

for ε>0. In this case, we have

$$\mu^{\alpha}_{\Delta_{(\lambda,\gamma)}}(\Omega((m,n,k),\gamma,\lambda))\leq\frac{1}{\varepsilon}\int_{\Omega((m,n,k),\gamma,\lambda)} f_{mnk}(|x_{m_sn_sk_s}-1|\Delta^{\alpha}(x_{m_sn_sk_s})$$
$$\frac{1}{\varepsilon}\int_{[(mnk)+\gamma-\lambda_{mnk}+(m_0n_0k_0)-1,((mnk)+\gamma)_{\mathbb{T}}]} f_{mnk}(||x_{m_sn_sk_s}-1|\Delta^{\alpha}(x_{m_sn_sk_s})).$$

Proof. This can be proved by similar in [16].

Theorem 3 Let α be a fractional order and a triple sequence

$X:T^3 \to R^3$ be a $\Delta_{(\lambda,\gamma)}^{\alpha}$- measurable function, l∈R and 0<p<∞, then the following statements are equivalent.

(i) $[\hat{W}_{\gamma p}]_{\mathbb{T}}^{f} \subset \hat{S}_{\mathbb{T}}^{f(\lambda,\gamma)}$

(ii) If a triple sequence X is (λ,γ) uniformly strongly p- summable to l, then

(iii) $\hat{S}_{\mathbb{T}}^{f(\lambda,\gamma)} - \lim_{mnk\to\infty} X_{mnk} = 1.$

and a triple sequence X is a bounded function, then the triple sequence X is uniformly strongly p- summable to l.

Proof (i) Let ε>0 and $[\hat{W}_{\gamma p}]_T^f$- $\lim_{mnk\to\infty} X_{mnk}$ = l. We can write

$$\int_{[(mnk)+\gamma-\lambda_{mnk}+(m_0n_0k_0)-1,((mnk)+\gamma)_{\mathbb{T}}]} f_{mnk}(|x_{m_sn_sk_s}-1|^p \Delta^\alpha(x_{m_sn_sk_s})) \geq$$
$$\int_{\Omega((m,n,k),\gamma,\lambda)} f_{mnk}(|x_{m_sn_sk_s}-1|^p \Delta^\alpha(x_{m_sn_sk_s})) \geq$$
$$\varepsilon^p \Delta_\gamma^\alpha(\Omega((m,n,k),\gamma,\lambda)).$$

Therefore,

$$[\hat{W}_{\gamma p}]_{\mathbb{T}}^{f} - \lim_{mnk\to\infty} X_{mnk} = 1 \Rightarrow \hat{S}_{\mathbb{T}}^{f(\lambda,\gamma)} - \lim_{mnk\to\infty} X_{mnk} = 1.$$

(ii) Let a triple sequence X be (λ,γ)- uniformly strongly p- summable to l. For given ε>0, let

$\Omega((m,n,k),\gamma,\lambda)=\{(m_sn_sk_s)\in[(mnk)+\gamma-\lambda_{mnk}+(m_0n_0k_0)-1,(mnk)+\gamma)\_T:(m_sn_sk_s)\in\Omega\}$

on time scale T. Then, it follows from Lemma 1 that

$$\varepsilon^p \mu\Delta_\gamma^\alpha(\Omega((m,n,k),\gamma,\lambda)) \leq$$
$$\int_{[(mnk)+\gamma-\lambda mnk+(m0n0k0)-1,((mnk)+\gamma)\mathbb{T})]} f_{mnk}(|x_{m_sn_sk_s}-1|^p \Delta^\alpha(x_{m_sn_sk_s})).$$

Dividing both sides of the last inequality by

$$\mu_{\Delta_{(\lambda,\gamma)}}^{\alpha}([(mnk)+\gamma-\lambda_{mnk}+(m_0n_0k_0)-1,((mnk)+\gamma)_{\mathbb{T}}])$$

and taking limit as m,n,k→∞, we get

$$\lim_{m,n,k}\to\infty \frac{\mu_{\Delta_{(\lambda\gamma)}^{\alpha}}(\Omega((m,n,k),\gamma,\lambda))}{\mu_{\Delta_{(\lambda\gamma)}^{\alpha}}([(mnk)+\gamma-\lambda_{mnk}+(m_0n_0k_0)-1,(mnk)+\gamma]_{\mathbb{T}})} \leq$$
$$\frac{1}{\varepsilon^p}\lim_{m,n,k}\to\infty \frac{1}{\mu_{\Delta_{(\lambda\gamma)}^{\alpha}}([(mnk)+\gamma-\lambda_{mnk}+(m_0n_0k_0)-1,(mnk)+\gamma]_{\mathbb{T}})} \times$$
$$\int_{([(mnk)+\gamma-\lambda_{mnk}+(m_0n_0k_0)-1,((mnk)+\gamma)_{\mathbb{T}}]} f_{mnk}(|x_{m_sn_sk_s}-1|^p \Delta^\alpha(x_{m_sn_sk_s})) = 0$$

which yields $\hat{S}_T^{f(\lambda,\gamma)} - \lim_{mnk\to\infty} X_{mnk} = 1.$

(iii) Let a triple sequence X be bounded and statistically convergent to l on T then, there exists a positive number M such that $|X_{m_sn_sk_s}| \leq M \forall (m_sn_sk_s) \in \mathbb{T}$ and also

$$\lim_{m,n,k\to\infty} \frac{\mu_{\Delta_{(\lambda\gamma)}^{\alpha}}(\Omega((m,n,k),\gamma,\lambda))}{\mu_{\Delta_{(\lambda\gamma)}^{\alpha}}([(mnk)+\gamma-\lambda_{mnk}+(m_0n_0k_0)-1,(mnk)+\gamma]_{\mathbb{T}})} \quad (6.2)$$

where Ω((m,n,k),γ,λ) is as before. Since

$$\int_{[(mnk)+\gamma-\lambda mnk+(m_0n_0k_0)-1,((mnk)+\gamma)_{\mathbb{T}}]} f_{mnk}(|x_{m_sn_sk_s}-1|^p \Delta^\alpha(x_{m_sn_sk_s})) =$$
$$\int_{\Omega((m,n,k),\gamma,\lambda)} f_{mnk}(|x_{m_sn_sk_s}-1|^p \Delta^\alpha(x_{m_sn_sk_s})) +$$

$$\int [(mnk)+\gamma-\lambda mnk+(m0n0k0)-1,((mnk)+\gamma)_{\mathbb{T}}] \setminus \Omega((m,n,k),\gamma,\lambda) f_{mnk}(|x_{m_sn_sk_s}-1|^p \Delta^\alpha(x_{m_sn_sk_s})) \leq$$
$$(M+|1|)^p \int_{\Omega((m,n,k),\gamma,\lambda)} f_{mnk}[\Delta^\alpha(x_{m_sn_sk_s})] +$$
$$\varepsilon^p \mu_{\Delta_{(\lambda,\gamma)}^{\alpha}}([(mnk)+\gamma-\lambda_{mnk}+(m_0n_0k_0)-1,(mnk)+\gamma]_{\mathbb{T}}).$$

We obtain

$$\lim_{m,n,k\to\infty} \frac{1}{\mu_{\Delta_{(\lambda,\gamma)}^{\alpha}}([(mnk)+\gamma-\lambda_{mnk}+(m_0n_0k_0)-1,(mnk)+\gamma]_{\mathbb{T}})} \times$$
$$\int_{[(mnk)+\gamma-\lambda_{mnk}+(m_0n_0k_0)-1,((mnk)+\gamma)_{\mathbb{T}}]} f_{mnk}(|x_{m_sn_sk_s}-1|^p \Delta^\alpha(x_{m_sn_sk_s}))$$
$$\leq (M+|1|^p \times \lim_{m,n,k\to\infty} \frac{\mu\Delta_{(\lambda\gamma)}^{\alpha}(\Omega((m,n,k),\gamma,\lambda))}{\mu\Delta_{(\lambda\gamma)}^{\alpha}([(mnk)+\gamma-\lambda_{mnk}+(m_0n_0k_0)-1,(mnk)+\gamma]_{\mathbb{T}})} + \varepsilon^p$$

(6.3)

Since ε is abitrary, the proof follows from (6.2) and (6.3).

Theorem 4 Let α be a fractional order and X be a triple sequence of Δ_γ^α- measurable function. Then

$$\hat{S}_{\mathbb{T}}^{f(\lambda,\gamma)} - \lim_{mnk\to\infty} X_{m_sn_sk_s} = 1 \Leftrightarrow \Delta_\gamma^\alpha -$$

measurable set Ω⊂T such that $\mathfrak{I}_{\mathbb{T}}^{\mathrm{a}}(\Omega)=1$ and $\lim_{mnk} X_{mnk}$ = l, (m,n,k)∈Ω((m,n,k),γ,λ).

Proof. It is similar way of Theorem 3.9 in [16].

Conclusion

In this study, introduced the triple sequence of statistical convergence, the concepts of γ and (λ,γ)- uniform density and uniform statistical convergence were defined on an arbitrary time scale. Defined γ- uniform Cauchy functions on a time scale also obtained some relations between these spaces.

Competing Interests

The authors declare that there is not any conflict of interests regarding the publication of this manuscript.

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